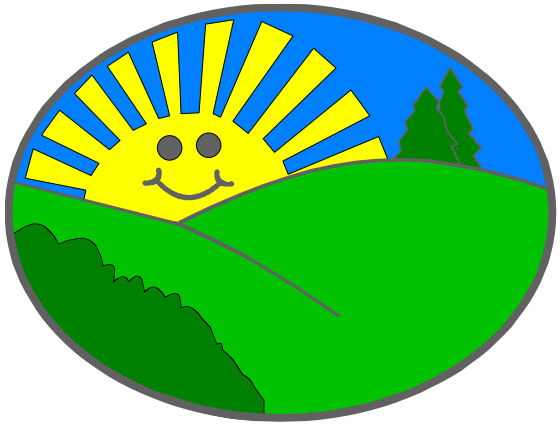
Shinfield Infant School

Concrete → Pictorial → Abstract

Maths Calculation Policy



This policy has been largely adapted from the White Rose Maths Hub Calculation Policy to be used alongside the schools teaching of White Rose Maths, supporting and challenging children's individual mathematical needs.

This policy outlines the children's learning development in stages: starting with **concrete** apparatus to learn mathematical concepts, then **pictorial** where pictures can be drawn to help solve problems and finally **abstract** where the children use the concrete and pictorial methods and apply this to abstract formats.

A range of models/resources are shown for each type of calculation. All these methods are taught to the children as laid out in the White Rose Maths program. This helps to develop children's *fluency* when working out calculations by providing them with a range of strategies they can use and embeds a concept by showing it in various ways; this is when a child is considered to have mastered a concept. Children who need challenge will be challenged at *greater depth* alongside each stage e.g. using abstract strategies alongside the concrete and pictorial stages.

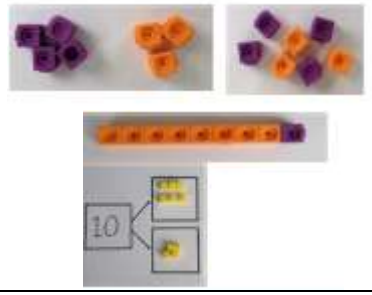
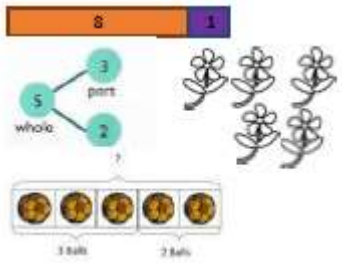
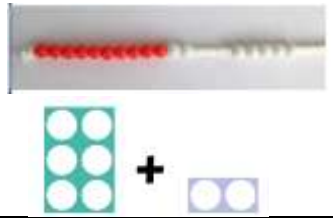
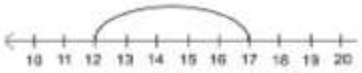
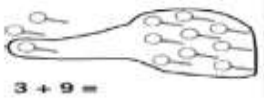
Various problem solving and reasoning skills are taught alongside the calculation methods so children can apply their calculation skills.

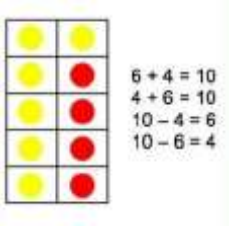
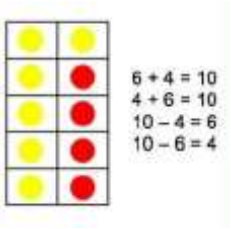
Video clips to demonstrate concepts and teaching methods are available on the website. The numbers are shown in green.

Year 1 Addition

within 10 and then 20

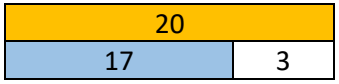
Use dienes or base ten (tens rods and ones/units cubes) for number in preparation for Year 2

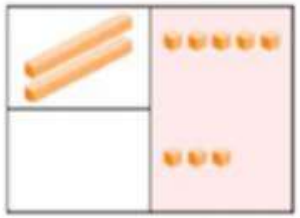
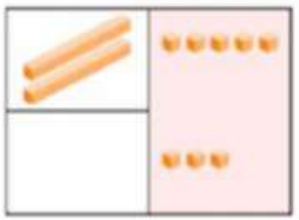
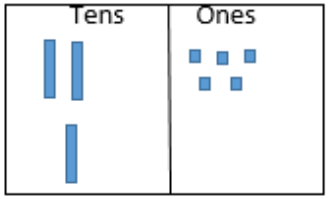
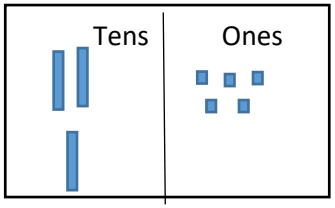
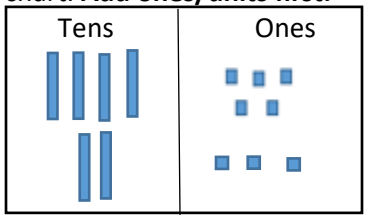
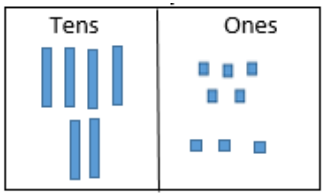
Objective/Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model Addition Year 1 set 1	Use part-whole model. Use cubes to add two numbers together as a group or in a bar. 	Use pictures to add two numbers together as a group or in a bar. 	Use the part-whole diagram as shown below to move into the abstract. Include missing number questions to support varied fluency. 8 = 5 + 3 5 + 3 = 8  8 = ? + 3 5 + ? = 8
Starting at the bigger number and counting on Addition Year 1 set 2	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. Use counters, tens frames or Numicon. 	Start at the larger number on the number line and count on in ones or in one jump to find the answer.  **12** + 5= 17	Place the larger number in your head and count on the smaller number to find your answer. 5 + **12** = 17
Regrouping to make 10 This is an essential skill for column addition later Addition Year 1 set 3	6 + 5 = 11 Start with the bigger number and use the smaller number to make 10. Use ten frames. 	Use pictures or a number line. Regroup or partition the smaller number using the part-whole model to make 10.  3 + 9 =	7 + 4= 11 3 +1 I am at seven, how many more do I need to make 10? (3) How many more do I add on now? (1)

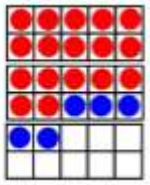
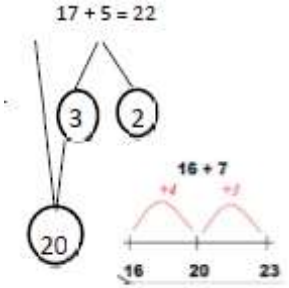
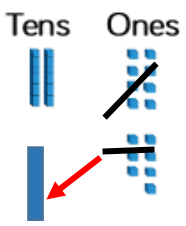
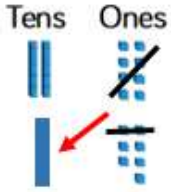
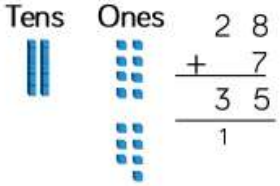
	9+3=12 This example shows how bead strings can be used to demonstrate the same method. 		
Use number bonds to 10 to make number bonds to 20 Addition Year 1 set 4	Use 10s frames and coloured counters (1 frame for the number bond to 10 and 2 frames for the number bond to 20) and Numicon. 	Colour in dots (2 different colours to make the bonds) on 10s frames (1 frame for the number bond to 10 and 2 frames for the number bond to 20). 	7+3=10 so 17+3=20 or 13+7=20

Year 2 Addition

up to 100 and starting to cross over 100

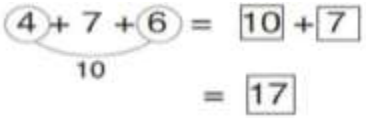
Objective/Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten to make 100 and numbers up to 100. Addition Year 2 set 5	Model creating number bonds using a 10s frame to represent 100 and 2 different coloured counters, dienes or Numicon 10s.  (Each counter represents 10 in this example) 60 + 40 = 100 	Drawing of 10s rods, 10s numberline or 10s place value counters on a 10s frame to represent 100.  60 + 40 = 100	20 + 30 = 50 70 = 50 + 20 40 + □ = 60 Recite number bonds to 100
Use known addition number facts to make 20 to explore **subtraction** facts (to make 20 and numbers up to 20) links to Year 2 subtraction Addition Year 2 set 6	Children explore ways of making numbers within 20 using part-whole model, counters or cubes and 2 tens frames with 2 different colour counters. 	Use bar model, number lines, part-whole model with numbers, drawings of 10s frames with dots.   	Explore commutativity of addition and make list. E.g. 12+3=15 15=12+3 3+12=15 15=3+12 Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. E.g. 12+3=15 So 15-3=12 and 15-12=3

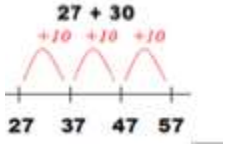
Add a two-digit number and ones (not bridging 10s, so no exchanging) Addition Year 2 set 7 See also Empty Number line method as an alternative (below)	Use dienes and place value chart**. Add ones/units first.** 	Use part-whole model or number track to model. When bridging 10s use a number line, draw dienes in place value chart. Add ones/units first. 	Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values. 
Add a 2-digit number and tens Addition Year 2 set 8	25 + 10 = 35 Explore that the ones digit does not change. Use dienes in a place value chart. **Add ones/units first.** 	Draw dienes in the place value chart to help to lead into column addition. Look at ones/units column first to see if there's anything to add.  25 + 10 = 35	27 + 10 = 37 27 + 20 = 47 27 + ☐ = 57
Add two, 2-digit numbers (no exchanging) Addition Year 2 set 9	45 + 23 = Use dienes in a place value chart. **Add ones/units first.** 	Draw dienes in the place value chart to help to lead into column addition. Add ones then add tens. 	27 + 31 = 58 31 + ☐ = 58

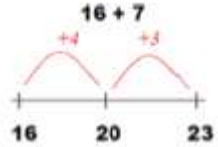
	Add the ones then add the tens.		
Regrouping to make 10 *This is an essential skill for column addition* Addition Year 2 set 10	Start with the bigger number and use the smaller number to make 10. Use ten frames.  17 + 5 = 22 17 + 3 = 20 20 + 2 = 22 Explore the pattern 17 + 5 = 22 27 + 5 = 32	Regroup or partition the smaller number using the part-whole model to make 10, a number line, 10s frames. 	Regroup to make other multiples of 20 + E.g. 33+9= 33+7+2=
Add two numbers **exchanging** 1s for 10s Addition Year 2 set 11	Model using dienes on a place value chart, exchange ten 1s for a tens rod and move that with the 10s. **Add ones/units first.** 28+7= 	Model drawing dienes on a place value chart, exchange ten 1s for a tens rod and move that with the 10s by crossing out the ten 1s and drawing the exchanged 10 under the tens column. 28+7= 	Use column addition format. 

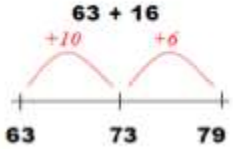
<table>
<tr>
<td>Add three 1-digit Numbers

Addition Year 2 set 12</td>
<td>Combine to make 10 first if possible, or bridge 10 then add third digit. Use 3 different coloured counters, cubes.
  </td>
<td colspan="2"> 

 
</td>
<td>Combine the two numbers that make/ bridge ten then add on the third.
 
</td>
</tr>
<tr>
<td>Add with an empty number line

Addition Year 2 clips 13, 14 & 15</td>
<td colspan="2">Add multiples of 10
 

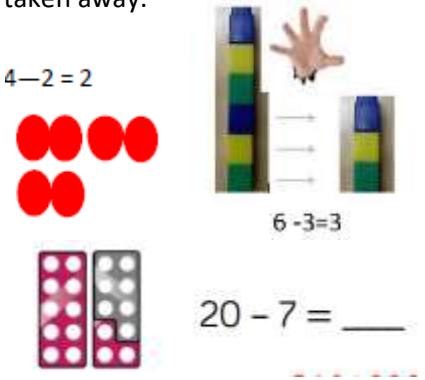
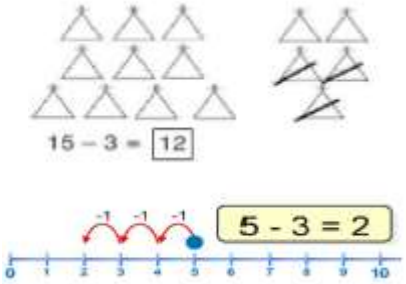
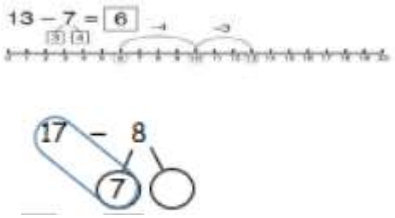
Add 1-digit numbers
 

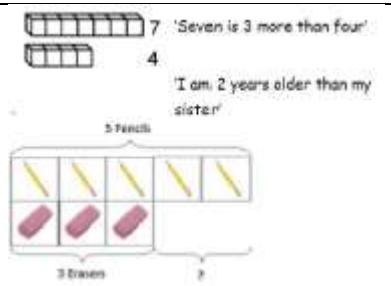
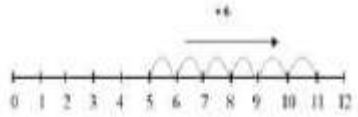
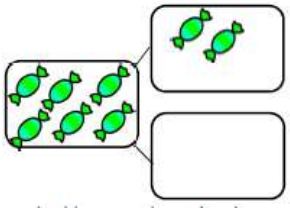
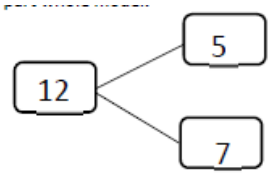
Add pairs of 2-digit numbers
 
</td>
<td>For concrete support use a 100 square alongside or dienes.

For the pictorial stage, add on ready drawn number lines.</td>
<td>Independently use the empty number line method (apparatus can still be used alongside).</td>
</tr>
</table>

Year 1 Subtraction

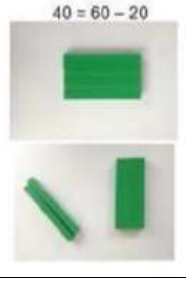
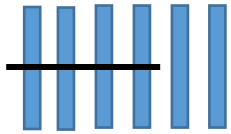
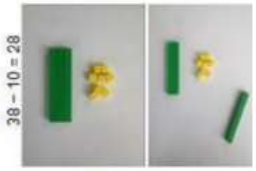
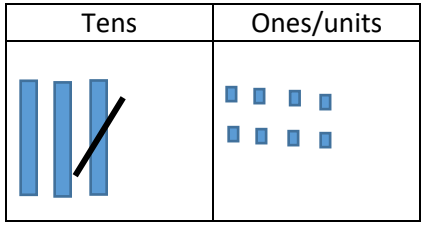
Use dienes (tens and ones/units) for number in preparation for year 2

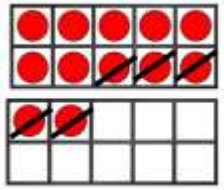
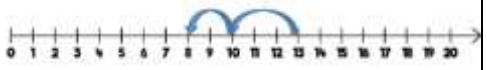
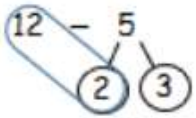
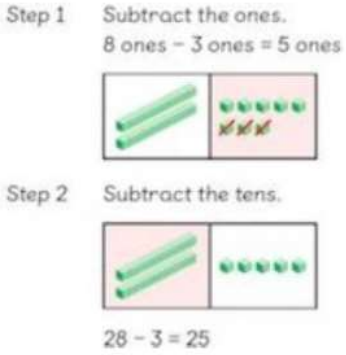
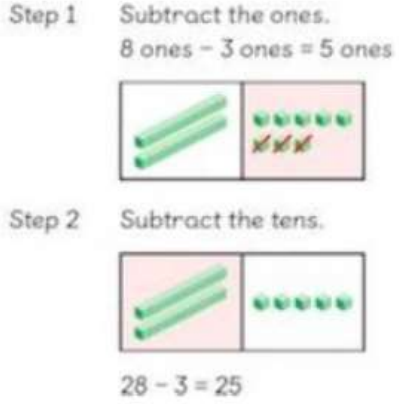
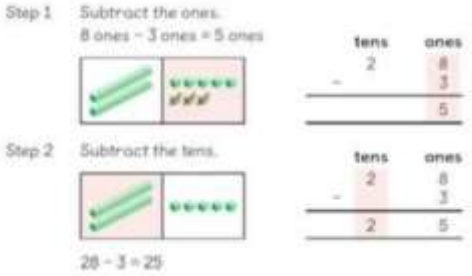
Objective/Strategy	Concrete	Pictorial	Abstract
Taking away 1-digit then 2-digit numbers up to 20 (start by not crossing 10, then crossing 10) Subtraction Year 1 set 1	Use physical objects, counters, cubes, bead strings, Numicon, etc. to show how objects can be taken away. 	Cross out drawn objects to show what has been taken away, count back on a number line, bar model. 	7—4 = 3 16—9 = 7 Put 13 in your head, count back 4. What number are you at?
Make 10 when counting back to cross over 10 Subtraction Year 1 set 2	14—5= Make 14 on the ten frame with counters. Take 4 away to make ten, then take one more away so that you have taken 5. 	Use a number line, jump back 3 first, then another 4. Use ten as the stopping point. Use part-whole model. 	16—8 How many do we take off first to get to 10? How many left to take off?
Find the difference (relate to addition, counting on as well as counting back)	Compare objects and amounts Lay objects to represent bar model.	Count on using a number line to find the difference.	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?

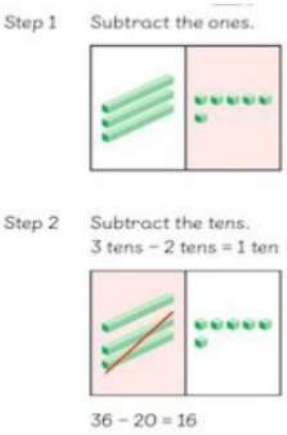
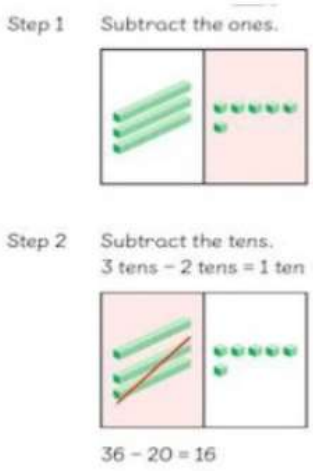
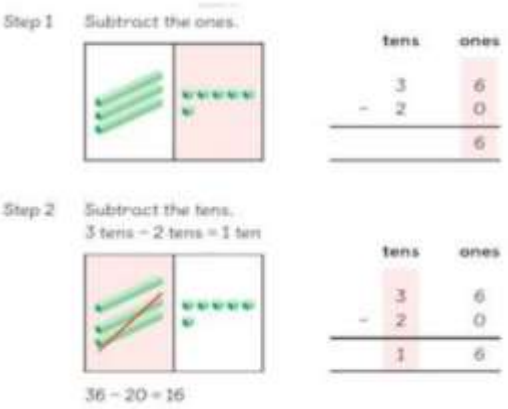
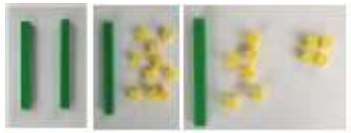
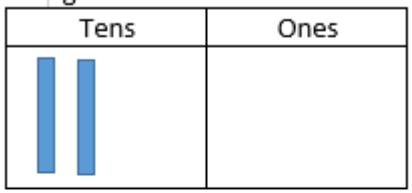
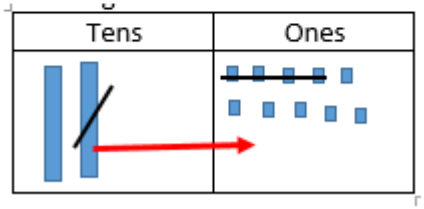
Subtraction Year 1 set 3			
Represent and use number bonds and related subtraction facts within 20. Include subtracting zero. Part-whole model Subtraction Year 1 set 4	Link to addition. 12+1=13 13-1=12 Use 10s frames and 2 different coloured counters to model inverse.  If 10 is the whole and 6 is one of the parts, what is the other part? 10 – 6 = 4	Use part-whole model (dienes drawn), bar model and draw dots in 10s frames. 	Move to using numbers within the part-whole model. Include missing number problems: 12 - ? = 5 7 = 12 - ? 

Year 2 Subtraction

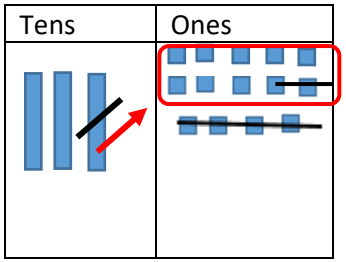
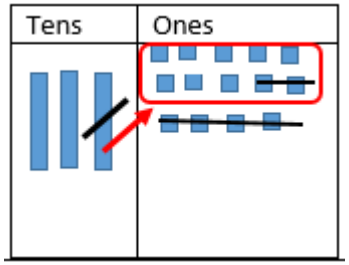
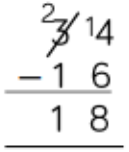
Subtraction number facts included in Year 2 Addition

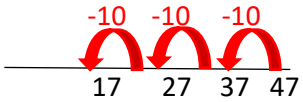
Objective/Strategy	Concrete	Pictorial	Abstract
Subtracting multiples of ten to make 100 and numbers up to 100 Subtraction Year 2 set 5	Model using a 10s frame to represent 100 and 2 different coloured counters to create number bonds, dienes or Numicon 10s. 	Drawing of 10s rods and cross out 10s, 10s numberline or 10s place value counters on a 10s frame to represent 100. 	30 - 20= 10 70 = 100 - 30 90 - ☐ = 60 Recite subtraction number bonds to 100
Subtract multiples of 10 from numbers to 100 Subtraction Year 1 set 6 See also Empty Number line method as an alternative (below)	Use a place value chart with dienes (start to lead into column subtraction). **Subtract ones/units first.** 38 – 10 = 	Use a place value chart and draw dienes (start to lead into column subtraction). 38 – 10 = 	Relate place value chart with dienes to column subtraction. 3 8 - 1 0 _____ _____
Make 10 when counting back to cross over 10 Subtraction Year 1 set 7	Use 10s frames to subtract back to the 10 by partitioning the second number. 12 -5=	Use number line and part-whole model to subtract back to the 10 by partitioning the second number.	Use the strategy mentally e.g. 23 – 5 = so 23 – 3 = 20, then there's 2 left to subtract (because 3 + 2 = 5) so 20 – 2 = 18

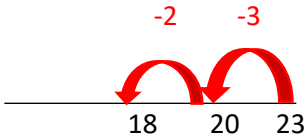
	 12 – 2 = 10 10 – 3 = 7	13 – 5 =  12 – 5 = 	
Subtract a 1-digit number from a 2-digit (no exchanging) Subtraction Year 1 set 8	Use dienes, subtract by moving ones away. **Subtract ones/units first.** 	Use drawings of dienes in a place value chart and cross out to subtract. 	Start using numerical column subtraction. 

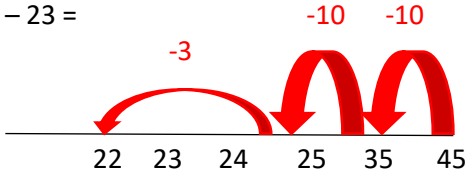
Subtract two 2-digit numbers (no exchanging) Subtraction Year 1 set 9	Use dienes, subtract by moving ones/units away. **Subtract ones/units first.** 	Use drawings of dienes in a place value chart and cross out to subtract. 	Start using column subtraction. 
Regroup 10 into 10 ones (to prepare for exchanging) Subtraction Year 1 set 10	Use a place value chart to show to change a 10 into 10  ones.	Draw a place value chart to show to change a 10 into 10 ones.  20 – 4 =  Use ‘exchange’ and ‘take and make’.	20 – 4 = 16

<table>
<tr>
<td>Subtract two 2-digit numbers (exchanging 10) Included in Year 2

Subtraction Year 1 set 11</td>
<td>Use dienes, exchange a 10 for ten 1s/units. Subtract ones/units first.
34 – 16 =
 
</td>
<td>Use drawings of dienes in a place value chart and to exchange 10, cross out 10s rod and draw ten ones then cross out amount to subtract.
 
</td>
<td>Start using column subtraction.
 
</td>
</tr>
<tr>
<td>Subtract with an empty number line

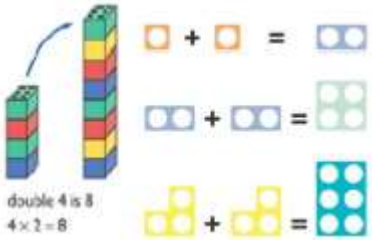
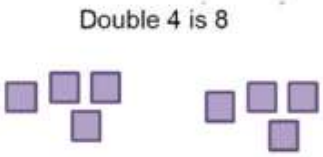
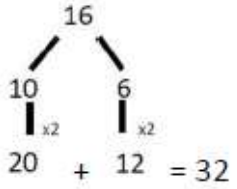
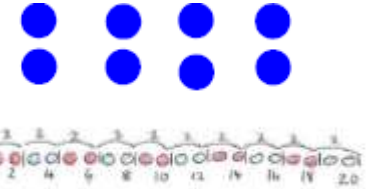
Subtraction Year 1 clips 12, 13 & 14</td>
<td colspan="2">Subtract multiples of ten
47 – 30 =
 

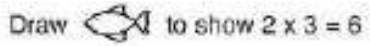
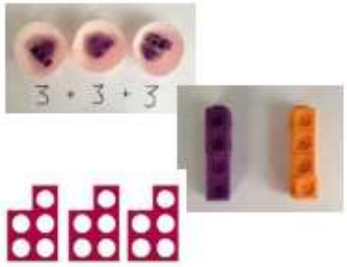
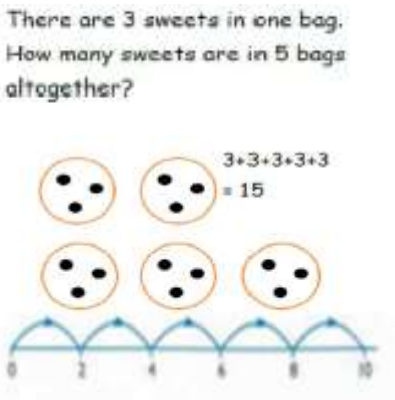
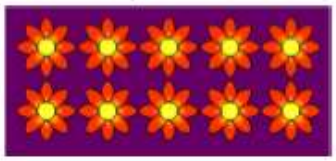
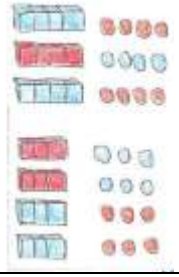
Subtract 1-digit numbers
23 – 5 =
 

Subtract pairs of 2-digit numbers
45 – 23 =
 

For concrete support use a 100 square alongside or dienes.

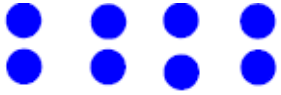
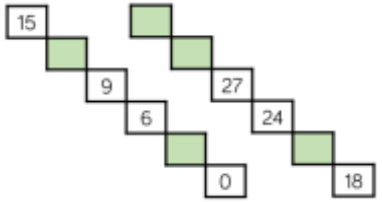
For the pictorial stage, subtract on ready drawn number lines.</td>
<td>Independently use the empty number line method (apparatus can still be used alongside).</td>
</tr>
</table>

Year 1 Multiplication			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Doubling Multiplication Year 1 set 1	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.  	Draw pictures to show how to double numbers.  	Partition a number and then double each part before recombining it back together. 
Counting in multiples (2s, 5s, 10s) Multiplication Year 1 set 2	Count the groups of 2, 5 or 10 using bead strings, number lines, 100 square, Numicon, looking at images of groups. Children could use their fingers as they are counting. 	Children draw representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

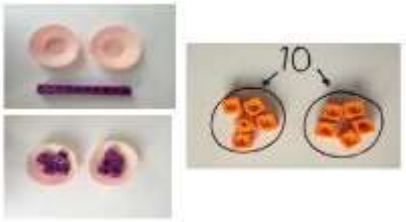
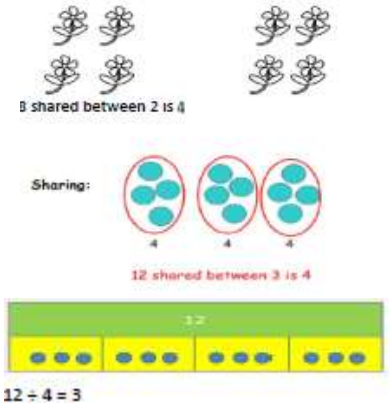
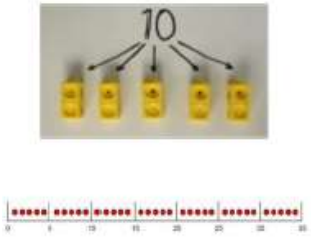
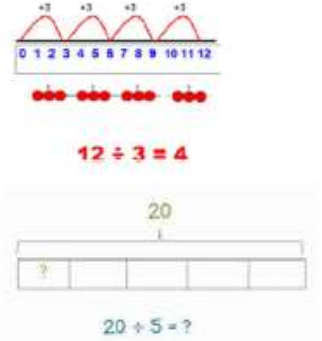
Making equal groups and counting the total Multiplication Year 1 set 3	Use different objects to add equal groups. 	Draw and make representations. 	4x2=8
Repeated addition Multiplication Year 1 set 4	Use different objects to add equal groups e.g. Numicon, cubes, counters, bead strings. 	Draw pictures to show solutions. Jump in steps on a number line. 	Write addition sentences to describe objects and pictures. 
Understanding Arrays Multiplication Year 1 set 5	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding. 	3 x 2 = 6 2 x 3 = 6

Year 2 Multiplication

Children should be able to recall and use the multiplication and division facts for 2, 5 and 10 x table
Links between multiplication and division in the division section

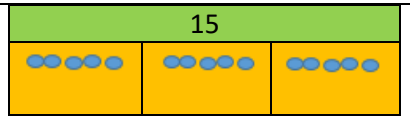
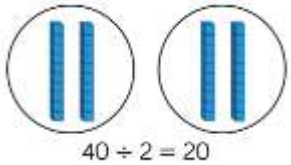
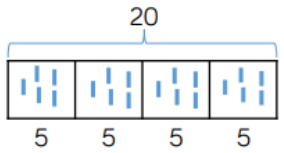
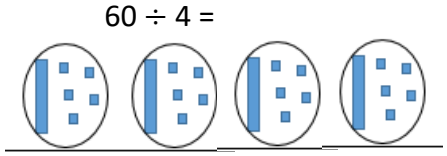
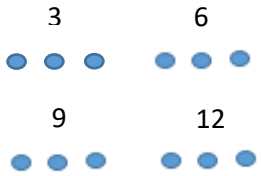
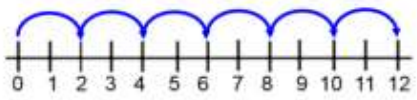
Objective/Strategy	Concrete	Pictorial	Abstract
Counting in multiples of 3 (recap 2, 5 and 10) Multiplication Year 2 set 6	Count the groups of 2, 5, 10 and 3s using bead strings, number lines, 100 square, Numicon, looking at images of groups. 	Draw number lines counting in groups for hops.  	Count in multiples of a number aloud. Write sequences with multiples of numbers (fill in missing numbers from pattern). 0, 3, 6, 9, 12, 15 
Using arrays to solve multiplication calculations Multiplication Year 2 set 7	Use objects including dienes laid out in arrays to find the answers to 3 x 5, 3 x 12 etc.  	Draw representations of arrays to solve multiplication calculations.  	Write different calculations for an image of an array including + and x.  5 + 5 + 5 + 5 = 20 4 + 4 + 4 + 4 + 4 = 20 4 x 5 = 20 5 x 4 = 20

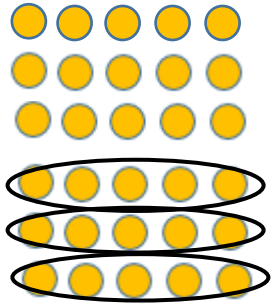
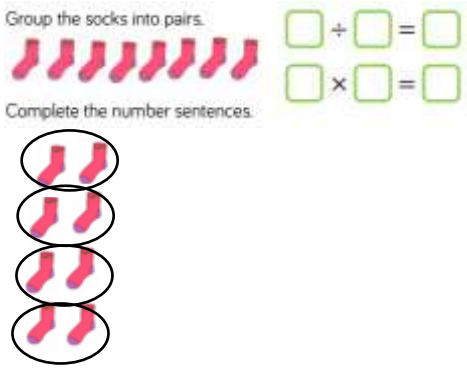
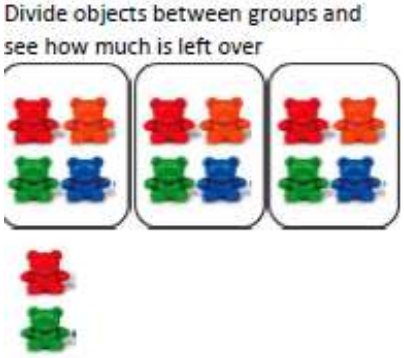
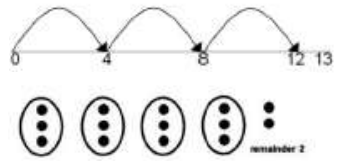
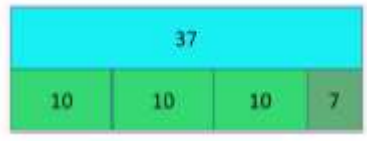
Multiplication is commutative Multiplication Year 2 set 8	Create arrays using counters and cubes and Numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  3 x 5 = 15  5 x 3 = 15	Use representations of arrays to show different calculations and explore commutativity.  3 x 5 = 15 15 = 3 x 5  5 x 3 = 15 15 = 5 x 3	12 = 3 × 4 so 12 = 4 × 3

Year 1 Division			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Division as sharing (children do not need to be familiar with the symbol yet) Division Year 1 set 1	I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures and bar models to share quantities. E.g draw pictures of sharing 10 muffins between 2 plates. 	12 shared between 3 is 4. (12 ÷ 3 = 4)
Introduce division as grouping (children do not need to be familiar with the symbol yet) Division Year 1 set 2	Divide quantities into equal groups e.g. 20 counters in total how many equal groups of 2 can you make? Use cubes, counters, objects or place value counters to aid understanding. 	If you have a total of 12, how many equal groups of 3 will you have? Use a number line or a bar model. 	Divide 28 into 4s. How many equal groups do you get? (28 ÷ 4 =)

Year 2 Division

Children should be able to recall and use the multiplication and division facts for 2, 5 and 10 x table

Objective/Strategy	Concrete	Pictorial	Abstract
Division as sharing (with ÷ symbol) Division Year 2 set 3	 Use a bar model or draw groups to solve division calculations with counters, cubes or dienes. 	Draw a bar model/draw pictures to solve division calculations.   Children will need to exchange 2 tens for 20 ones/units so they can put 1 ten and 5 ones in each group.	How many different ways can you divide/share equally 20? 20 ÷ 1 = 20 20 ÷ 20 = 1 20 ÷ 2 = 10 20 ÷ 10 = 2 20 ÷ 4 = 5 20 ÷ 5 = 4
Division as grouping (with ÷ symbol) Division Year 2 set 4	Divide quantities into equal groups e.g. 12÷3= ; get 12 counters/cubes divide them into 3s. How many groups are there? 	Use bar modelling and a number line to aid solving division problems by grouping. 	How many groups of 4 in 24? 24 ÷ 4 = ? 24 ÷ ? = 4

Division with arrays (link division and multiplication- Inverse) Division Year 2 set 5	Link division to multiplication by creating an array using cubes, counters, numicon E.g. for 3 x 5 = 15 (3 rows of 5) Model how 15 ÷ 3 = 5 	Draw an array and use lines to split the array into groups to make multiplication and division Sentences. 	Find the inverse of multiplication and division sentences by creating eight linking number sentences E.g. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4
Division with remainders. **Greater depth Year 2** Division Year 2 set 6	Divide objects between groups and see how much is left over. 14 ÷ 3 = 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder. Use bar models to show division with remainders.  13 ÷ 4 = 3 remainder 1  37 ÷ 10 = 3 remainder 7	Complete written divisions and show the remainder using r.